# Nonlocal gravity 

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Refs: PRD 98, 084040 (2018), PRD 99, 064044 (2019)

2019/04/28@CCNU

- Motivation to modify gravity: The late-time acceleration
- General result to explain the acceleration


Maggiore etal, $P R D, 2014$

- Coincidence problem

The following may be a better theory:

- The late-time acceleration
- Newtonian potential Psi=Phi exactly
- Except kappa=8PiG/c^2, only dimensionless parameters appear in the Lagrangian
- Pure geometric theory


## For dimensionless parameters:

- Dimension analysis: $[R]=$ length $\wedge\{-2\}$
- If we use $\mathrm{R}^{\wedge} 2$, then we need length^2* $\mathrm{R}^{\wedge} 2$
- The coefficient length^2 can be a parameter (c/HO etc), or a math operator (Box^\{-1\}, an integral operator)
- Box^\{-1\} $\rightarrow$ nonlocal gravity

History of nonlocal gravity:

- Deser \& Woodard, PRL, 2007 (not Psi=Phi)

$$
\Delta \mathcal{L} \equiv \frac{1}{16 \pi G} R \sqrt{-g} \times f\left(\frac{1}{\square} R\right),
$$

- Maggiore \& Mancarella, PRD, 2014 (not dimensionless)

$$
S_{\mathrm{NL}}=\frac{1}{16 \pi G} \int d^{d+1} x \sqrt{-g}\left[R-\frac{d-1}{4 d} m^{2} R \frac{1}{\square^{2}} R\right],
$$

- And so on, but none of them satisfy previous 4 requirements

One attempt (Tian, PRD, 2018)

$$
\mathcal{L}_{G}=\frac{1}{2 \zeta}\left(R_{\mu \nu} \square^{-1} R^{\mu \nu}-\frac{1}{2} R \square^{-1} R\right),
$$

- [zeta]=[kappa]
- $-1 / 2$ is for Psi=Phi, based on

$$
\begin{aligned}
& \mathrm{d} s^{2}=-c^{2}\left(1+2 \Phi / c^{2}\right) \mathrm{d} t^{2}+\left(1-2 \Psi / c^{2}\right) \mathrm{d} \mathbf{r}^{2}, \\
& \square^{-1} R^{\mu \nu}=0 \text { in the background }
\end{aligned}
$$

- This assumption has been used in: Koivisto, PRD, 2008; Maggiore \& Mancarella, PRD, 2014; Kehagias \& Maggiore, JHEP, 2014; Conroy etal, CQG, 2015; etc.


## How to set the background?

- Cosmological evolution may give the nonlocal field a nonzero value in the solar system scales.
- Belgacem, Finke, Frassino, and Maggiore, JCAP, 2019 proved this with the McVittie metric.
- And then, Deser2007 and Maggiore2014 (RR model) theories are ruled out by the time-varying G observations. (Belgacem2019)
theory: $\left|\dot{G}_{\text {eff }} / G_{\text {eff }}\right|=\mathcal{O}\left(H_{0}\right)$.
observations: $|\dot{G} / G|<10^{-12} \mathrm{yr}^{-1} \approx 0.01 \mathrm{H}_{0}$

Revisit Tian2018 model

$$
\begin{aligned}
\mathcal{L}_{G} & =\frac{1}{2 \zeta}\left(R_{\mu \nu} \square^{-1} R^{\mu \nu}-\frac{1}{2} R \square^{-1} R\right), \\
\mathrm{d} s^{2} & =-c^{2}\left(1+2 \Phi / c^{2}\right) \mathrm{d} t^{2}+\left(1-2 \Psi / c^{2}\right) \mathrm{d} \mathbf{r}^{2}, \\
U_{00} & =f_{10} c^{2}+f_{11}(\mathbf{r}) c^{2}, \quad U_{0 i}=0, \quad U_{\mu \nu} \equiv \square^{-1} R_{\mu \nu} \\
U_{i j} & =\delta_{i j} f_{20}+\delta_{i j} f_{21}(\mathbf{r}),
\end{aligned}
$$

- Results:

Psi=Phi

$$
\begin{aligned}
& G_{\text {eff }}=\frac{\zeta c^{4}}{8 \pi\left[-\left(f_{10}+f_{20}\right)^{2}+f_{10}-f_{20}+1\right]}, \\
& \left|\dot{G}_{\text {eff }} / G_{\text {eff }}\right|=\mathcal{O}\left(\dot{H}_{0}\right) .
\end{aligned}
$$

Newtonian approximation in nonlocal Gauss-Bonnet gravity

Capozziello etal, PLB, 20

$$
\begin{aligned}
& \mathcal{L}_{G}=\frac{1}{2 \kappa}\left(R-\alpha \mathcal{G} \square^{-1} \mathcal{G}\right), \quad \text { [alpha]=length^4 } \\
& \mathcal{G}=R^{2}-4 R_{\mu \nu} R^{\mu \nu}+R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma} . \\
& \mathrm{d} s^{2}=-c^{2}\left(1+2 \Phi / c^{2}\right) \mathrm{d} t^{2}+a^{2}\left(1-2 \Psi / c^{2}\right) \mathrm{d} \mathbf{r}^{2}, \\
& U=U_{0}(t)+U_{1}(\mathbf{r}, t), \quad U \equiv \square^{-1} \mathcal{G}
\end{aligned}
$$

$H_{0} / c \approx \partial /(c \partial t) \ll \partial /(a \partial x)$

$$
\begin{gathered}
\gamma \equiv \frac{\Psi}{\Phi}=\frac{c^{4} /(8 \alpha)+8 H^{4}+8 H^{2} \ddot{A}+C_{1} H / a^{3}}{c^{4} /(8 \alpha)-24 H^{4}+24 H^{2} \ddot{A}+16 \ddot{A}^{2}-3 C_{1} H / a^{3}}, \quad \ddot{A} \equiv \ddot{a} / a . \\
G_{\text {eff }}=\frac{\kappa c^{4}}{4 \pi\left(\gamma r_{1}+r_{2}\right)} . \begin{array}{r}
r_{1}= \\
\frac{2 H^{2}}{\ddot{A}}+2-\frac{48 \alpha C_{1} H^{3}}{\ddot{A} a^{3} c^{4}}+\frac{384 \alpha H^{2} \ddot{A}}{c^{4}}+\frac{16 \alpha C_{1} H}{a^{3} c^{4}} \\
\quad-\frac{384 \alpha H^{6}}{\ddot{A} c^{4}}+\frac{512 \alpha H^{4}}{c^{4}}, \\
r_{2}=-\frac{2 H^{2}}{\ddot{A}}-\frac{16 \alpha C_{1} H^{3}}{\ddot{A} a^{3} c^{4}}-\frac{128 \alpha H^{6}}{\ddot{A} c^{4}}-\frac{192 \alpha H^{4}}{c^{4}} .
\end{array}
\end{gathered}
$$

- Generally, $\left|\dot{G}_{\text {eff }} / G_{\text {eff }}\right|=\mathcal{O}\left(H_{0}\right) . \quad \Psi \neq \Phi$.
- In the de Sitter phase, Psi=Phi and G_eff is constant.


## Summary

- The RT model (Maggiore, PRD, 2014) is the only viable nonlocal gravity theory when confronting observations for now. (Belgacem etal, JCAP, 2019; Tian \& Zhu, PRD, 2019)
- Nonlocal gravity may cannot achieve the initial four requirements (Psi=Phi, dimensionless...), and may cannot solve the coincidence problem.

Thanks

